Study of gravity induced Particle Production in earth's gravitational field

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Abstract

In this paper the production of particles have been discussed via interaction with the earth's gravitational field. Explicit calculations have been done for high energy scalars passing through earth's gravitational field. Here it is shown that the width for the scalar process can become comparable with a typical weak decay width at an energy scale of few Tev. We have speculated that similar processes may be responsible for many of the anomalies in the $10–10^4$ Tev experimental data.

Key Words : - Cosmic ray, Weinberg-Salam model, $\lambda\phi^4$ theory, Higgs Scalar, Cygnus X-3

1. Introduction

Gravitational interactions are generally neglected in the elementary particle reactions, because it is generally believed that their effects are felt only at energy scales of the order of the Planck mass. The primary purpose of this work is to show that this need not be true, i.e. earth's gravitational

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field could produce significant effects in elementary particle reactions in the TeV range. The secondary purpose of this work is to speculate that similar processes may be responsible for many anomalies in the TeV physics such as cosmic ray data. Our explanations are then within the standard model and do not require new interactions[1].

We work mainly with the effect of gravitational interactions in scalar particle processes in $\lambda\phi^4$ theory, but our conclusions can be generalized to any process that can involve virtual scalars coupling to gravity as indicated later.

We have recently noted[2] that the matrix elements of the unique finite energy momentum tensor for the $\lambda\phi^4$ theory discovered by Collins[3] viz.

$$\theta_{\mu v} = \delta_\mu \phi \delta_v \phi - \eta_{\mu v} \varphi + \frac{g(\varepsilon)}{1-n} (\delta_\mu \delta_v - \eta_{\mu v} \delta^2) \phi^2 \qquad \text{.............(1)}$$

have an unusual high energy behaviour. It was shown[2] that under certain conditions, this $\theta_{\mu v}$ behaves as an operator with an effective anomalous dimensions $2\gamma_m$ (λ*) While the result[3] was derived for $\lambda\phi^4$ theory which, may turn out to be trivial, similar results have been derived for gauge theories with scalars, e.g. the Weinberg-Salam model. Therefore the results in this work hold in a more general context even though we shall continue, for concreteness and simplicity, to talk in the context of $\lambda\phi^4$ theory. If, thus

$2\gamma_m$ (λ*) is large and positive it could produce significant large gravitational effects.

In view of the fact that our results are based on the above observation we shall, very briefly, state how this comes about. We define the coefficient of the simple pole terms in (n-2-4g(ε))$Z_m^{-1}\equiv$ K(λ).

Here Z_m is the renormalization constant appearing in the mass renormalization $mo^2=m^2 Z_m^2$. K(λ) satisfies a differential equation and from this it was deduced that, K(λ) can blow up near a nontrivial fixed point $\lambda = \lambda^*$ of the theory assumed to exist. Now when one considers the truncated n-point Green's function of $\theta_{\mu v}$ viz. $G_{\mu v}^{(n)t}$, it can be expressed as a series of Green's functions of six linearly independent operators[4]. The coefficients of five others are O^1 while that of the operator.

$O_8 = (\delta_\mu \delta_v - \delta^2 \eta_{\mu v})\phi^2$

is

$-\beta(\lambda)/[24\gamma_m(\lambda)]\, dK(\lambda)/d\lambda$;

and as shown[2], this can also blow up at $\lambda = \lambda^*$ (despite the fact that $\beta(\lambda^*)=O$). Thus this last term dominates near $\lambda = \lambda^*$ and one has[4].

$$G_{\mu v}(n)t \simeq -\frac{\beta(\lambda)}{24\gamma_3(\lambda)}\frac{dk(\lambda)}{d\lambda}G_8(n)t = \frac{\beta(\lambda)}{24\gamma_m(\lambda)}\frac{dk(\lambda)}{d\lambda}(q_\mu q_v - \eta_{\mu v}q^2)G_7^{(n)t} \quad ......(2)$$

Here $G_7^{(n)t}$ refers to the Green's functions of $O_7 = \phi^2(x)$ and q refers to the 4-momentum entering via O_7. O_7 is an operator of anomalous dimension $2\gamma_m$ and this leads[4] near a fixed point, to

$$G_{\mu\nu}^{(nt)}(exp(t)p_i, \lambda, m, \mu) \simeq -\frac{\beta(\lambda)}{24\gamma_m(\lambda)} - \frac{dk}{d\lambda} exp\{4 - n + 2\gamma_m(\lambda *)t\}$$

$$(q_\mu q_\nu - \eta_{\mu\nu} q^2) \times \; G_7^{(n)t}(p_i, \lambda *, \overline{m}(t) exp(-t), \mu) \qquad ....(3)$$

where the running coupling constant λ(t) and running mass parameter $\overline{m}(t)$ have been defined as usual[5]. The extra scaling factor exp $\{2\gamma_m(\lambda^*)t\}$ is a peculiarity of this energy-momentum tensor and the results[4] and in this work are a result of this factor.

In passing, we emphasize the significance of the of $\theta_{\mu\nu}$ of eq. (1). As shown in[6], this is the unique finite $\theta_{\mu\nu}$ for $\lambda\phi^4$ theory, which does not need an extra experimental input for its definition; thus if a principle of minimal coupling is adopted this $\theta_{\mu\nu}$ is the most likely candidate for the coupling of scalars to an external gravity. We assume this to be the case, here. In an earlier work[4], we explored the consequences of eq. (3) near a black hole and a neutron star. It was shown, for example, the gravitational corrections to the scattering of two scalars near a black hole could become comparable to the strong interaction cross-section at energies of the order of 25 TeV. This result, though an interesting one, is removed from applications that can be tested.

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In this paper we make an observation that is literally down to earth i.e. one that can in fact be tested in earth's gravitational field and that too at not too high an energy scale of say a few TeV. In the end, we speculate that many of the anomalies in TeV physics[7] may indeed be due to this effect.

2. MATHEMATICAL FORMULATION :

For concreteness we shall consider the process

$\phi(p_1) \rightarrow \phi(q_1) + \phi(q_2) + (q_3)$

in which the earth's gravitational field injects a 4-momentum

$q = q_1 + q_2 + q_3 - p_1.$

We shall work with a metric in which the earth's gravitational field is time independent so that $q_0 = 0$ We quantize the system in a volume V a straightforward calculation shows that the width for this process is

$$\Gamma_G = \frac{1}{384(2\pi)^8 V} \int \frac{q_1^2 q_2^2 dq_2 q_3^2 dq_3 d\Omega_1 d\Omega_2 d\Omega_3}{p_{10} q_{10} q_{20} q_{30}} 0 \, (p_{10} - p_{20} - p_{30})$$

$$x\theta(q_{20})\theta(q_{30})|h_{\mu v}(q)\, G_{\mu v}^{(4)t}(p_1, q_1, q_2, q_3, \lambda, m, \mu)|^2 \qquad ....(4)$$

Where

$g_{\mu v} = \eta_{\mu v} + h_{\mu v}$

and

$\bar{h}_{\mu v}(q) = \int v d^3x \exp(-iqx) h_{\mu v}(x)$

We use the Schwartzchild form of the metric[8]. Then, as shown[4], for $qr_o >> 1$ (where $r_0 = 2gm_e$), one obtains.

$$(q^\mu q^v - \eta^{\mu v} q^2) h_{\mu v}(q) = 4\pi r_0 [\cos(qr_2) - \cos(qr_1)] \qquad ....(5)$$

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where we have used the quantization volume as the one between concentric spherical shells with radii r_2, r_1 ($r_2 > r_1 > R_e$). It can be easily shown that as long as the linear dimensions of the quantization volume are >> 1 |q|, any quantization volume gives a result that does not alter the value of $|h_{\mu v}(q)\ G_{\mu v}^{(4)t}|^2$ substantially.

Using equations (2) and (5) we get

$$\Gamma_G \simeq \frac{(4\pi r0)^2}{24(2\pi)^8 V}\left|\frac{\beta(\lambda)}{96\lambda_m(\lambda)\lambda}\ \frac{dk\ (\lambda)}{d\lambda}\right|^2 \int \frac{q_1^2 q_2^2 dq_2 q_3^2 dq_3 d\Omega_1 d\Omega_2 d\Omega_3}{p_{10} q_{10} q_{20} q_{30}}$$

$$\theta(q_{20}) x \theta(q_{30})\ \theta(p_{10}-q_{20}-q_{30})\ [\cos(qr_2) - \cos(qr_1)]^2$$

$$x|G7^{(4)}(p_1, q_2, q_3, \lambda*, m, \mu)|^2 \qquad ....(6)$$

To study the behaviour of Γ_G with energy scale, we imagine rescaling all these momenta by exp(t). For massive on-shell scalars, the energy will not be scaled exactly be exp(t). Equation (3), on the other hand, relates the Green's functions in which the overall 4-momentum is scaled by exp(t). Thus, the use of eq. (3) for on shell Green's functions requires an extrapolation already discussed[4]. Further the scaling q→exp(t)q replaces the factor $[\cos(qr_2) - \cos(qr_1)]^2$ by $\cos\{\exp(t)qr_2] - \cos[\exp(t)\ qr_1\}^2$. As $qr_1 >> 1$, this factor is a very rapidly oscillating function, insensitive to t and can therefore be replaced by its average value 2. When this is done, one obtains.

$$\Gamma_G(t) = exp\,[4\gamma_m(\lambda*)t]\Gamma_G(0) \qquad ....(7)$$

Γ_G (0) may be negligible as compared to the weak and electromagnetic decay widths, but it can become comparable to these at high energies if γ_m (λ*) is large and positive enough. One has, on account of time dilation.

$\Gamma_{w/e.m.}$(t) = exp (-t) $\Gamma_{w/e.m.}$ (0) = exp (-t)$\frac{m_p}{\mu_0}$ $\Gamma_{w/e.m}$ (γ) ....(8)

where m_p is the physical mass; $\mu = \mu_0$ exp(t) and $\Gamma^{(\gamma)}$ is the decay width in the rest frame. Thus we have.

Γ_G (t)/$\Gamma_{w/e.m.}$ (t)= exp{[1 + 4γ_m (λ*)]t} Γ_G (0)/$\Gamma_{w/e.m.}$ (0) ....(9)

While this ratio is very small at ordinary energies, if γ_m (λ*) is large enough, it can become comparable to one in the TeV range.

In the following we shall make a crude estimate of Γ_G (0). In order that the term $G_8^{(4)t}$ dominates$G_{\mu v}^{(4)t}$, the coefficient [$\beta(\lambda)96\gamma_m(\lambda)$][dK/d$\lambda$] must be large, say 100, and let this happen at an energy scale $\mu_0 \simeq$100Gev. On dimensional grounds, we let $G_7^{(4)} \simeq 1/\mu^2$ where $\mu = p_0$ is the energy scale. We replace the oscillating factor $[\cos(qr_2)-\cos(qr_1)]^2$ by its average. We do the phase integral (for mp=0). We assume that the system is enclosed in a laboratory of volume V=10^5 m^3. Using r_0= 4.5×10^3m for earth, we find Γ_G(0) $\simeq$ 2.6 x10^{-2} s^{-1}. This is to be compared with a typical weak decay width (let mp=500MeV) ;

Γ_w (0) = 10^{10} S^{-1} (mp/μ_0)= 5 x 10^7 s^{-1}.This leads to

Γ_G(t)/Γ_w (t)$\simeq$ 5 x 10^{-10} exp{[1+4γ_m (λ*)]t} ....(10)

The energy scales at which this ratio becomes one is tabulated below:

$\gamma_m(\lambda^*)$	0.5	1	1.5	2	
μ(TeV)	126	7.2	2.18	1.09	...........(11)

In passing, we note the distinguishing characteristic of such a process: When a high energy particle interacting with say air molecule, most particles produced, would tend to go in the forward direction, whereas in the present case any three directions for the produced scalars are possible and could in particular lead to high p_1.

Next, we attempt at generalizations of the result. For concreteness, we had considered $\lambda\phi^4$ theory. One could for instance, consider the Weinberg-Salam model, and let ϕ be the Higgs scalar. The energy momentum tensor for such a theory generalizing that of (1) is obtained, where it is shown that the characteristic factor exp γ_m (λ^*)t is preserved in Γ_G = (t), and the result of (6) is expected to hold.

3. RESULTS AND DISCUSSION :

In this model itself, one could generalize the discussion to a process say a photon going into 3 photons. Here the photon could couple to scalars and the virtual scalars to gravity via the dominant term of the kind in (2). This would also lead to a similar expression for Γ_G^y (t), for this process in

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which $G_7^{(4)}$ refers to four photon Green's function. As a first approximation one would expect Γ_G^{y} (t)≈e^8 Γ_G^{ϕ}(t).

In general, any particle which couples to a scalar and the scalar to gravity via the dominant term of the kind in (2), would undergo similar processes with a similar enhancement with energy.

We considered, so far $\phi \rightarrow 3\phi$. In general, in the context of $\lambda\phi^4$ theory $\phi \rightarrow (2m+1)\phi$ is possible. It is easily seen from a generalization of (6) that phase space integral made dimensionless by dividing by appropriate power of p_{10} decreases rapidly as 1/(2m+3)! expecting a suppression for these processes. But then the number of diagrams contributing to $G_7^{(2m+2)}$ also increases rapidly.

Finally, we speculate on the possibility that some of the observed anomalies in the cosmic ray data in the PeV range may well be, due to this effect of gravitational interactions. Our analysis below is, by no means, meant to be thorough and exact, but rather suggestive and exploratory, keeping detailed analysis to a later work. As emphasized in the beginning, our explanations are entirely within the standard model.

We focus our attention on the data from Cygnus X-3 as compiled, selected and presented by Domokos et al[9] and on the data of Fenyves et al[10]. Domokos et al consider the hypothesis that all the primaries are photons, subject their data to analysis assuming a particular form for the photon

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spectrum and note that up to about 100 TeV their data fit this hypothesis but beyond this region there is a serious discrepancy. Fenyves et al[10] report a big jump in the number of photons from energy 100 TeV to 1000 TeV and a moderate jump from 10^3 TeV to 10^4 TeV[10]. We suggest that both enhancements are due to gravitational interactions; and we proceed to make an estimate of γ_m (λ^*) from these data on the assumption that these enhancements are entirely due to this effect.

Before proceeding, we make a general comment. Question arises as to what should one choose for V, the quantization volume, in (6). The only guide available is the result[4] that this volume is the volume over which the wavefunction of the particle is spread when it interacts with earth's gravitational field. As nothing is known about this, we have selected $V=10^5$m, which is subject to further investigation.

We analyse, roughly, the data presented by Domokos et al as follows. We find that a straight line fits the data well. Denoting I_{exp} to be the intensity observed experimentally fitted as above and I_{th} that obtained by Domokos et al on the basis of their hypothesis H_o, we find

$$\frac{I_{exp}}{I_{th}} \simeq 1 \; at 10^2 \text{ TeV}; \; \frac{I_{exp}}{I_{th}} \simeq 6.3 \text{ at } 10^3 \text{ TeV} \qquad ....(12)$$

In a similar manner, we find from the data of Fenyves et al

$$\frac{N\gamma(10^3 \text{TeV})}{N\gamma(10^2 \text{TeV})} \simeq 7.2. \qquad ....(13)$$

From (7), the enhancement from 10^2 TeV to 10^3 TeV depends solely on the exp $\{4\gamma_m(\lambda*)t\}$ factor. In order that there is no enhancement at 10^2 TeV and substantial enhancement by a factor of about 7 at 10^3 TeV, we set

$$\frac{L\Gamma_G}{c}\Big|_{10^3\,\mathrm{TeV}} \sim 5; \quad \frac{L\Gamma_G}{c}\Big|_{10^2\,\mathrm{TeV}} \sim 0.02; \quad (L^3 \equiv V) \qquad \text{.........(14)}$$

and we find that a value of $\gamma_m(\lambda^*) = 0.6$ would be enough to explain either enhancements and actual values.

One way to distinguish between the explanations based upon interaction of cosmic rays with air nuclei and on gravitational interaction is to conduct these experiments in a satellite where only gravitational interactions (somewhat weaker) would survive.

4. CONCLUSION:

We have discussed the production of particles via interaction with the earth's gravitational field. Explicit calculations have been done for high energy scalars passing through earth's gravitational field. We have shown that the width for the scalar process $\phi \rightarrow 3\phi$ can become comparable with a typical weak decay width at an energy scale of few TeV. We have speculated that similar processes may be responsible for many of the anomalies in the 10–10^4 TeV experimental data.

One final comment : As energy is further increased, the enhancement is not expected to increase as given by (7). This is because the processes involving more number of gravitational interactions become equally significant and from unitarity one knows that the net result cannot grow like a power of exp (t) = μ / μ_0. This may explain the lack of an enhancement by an equal factor beyond 10^3 TeV in results of Domokos et al and Fenyves et.al.

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